Max-Information, Differential Privacy, and Post-Selection Hypothesis Testing

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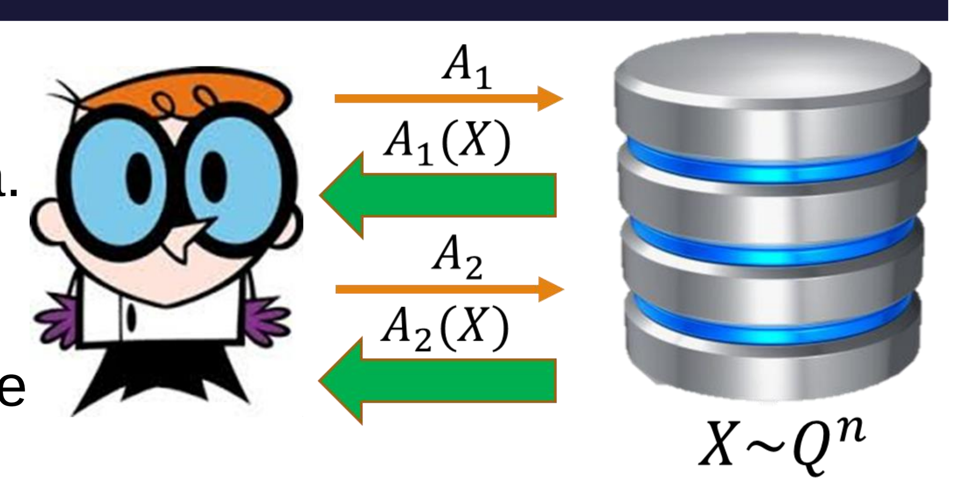
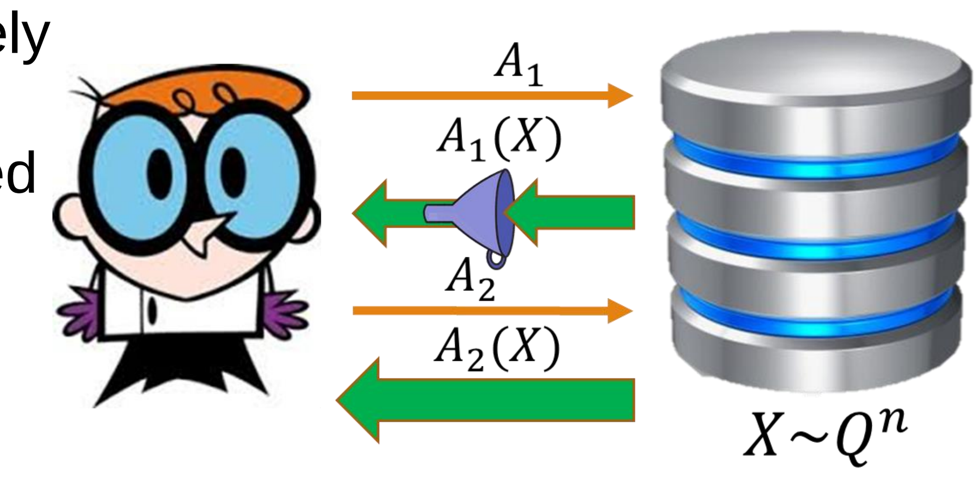
Adaptive Data Analysis refers to the reuse of data to perform analyses suggested by the outcomes of previously computed statistics on the same data. In this work, we initiate a principled study of how the generalization properties of *approximate differential privacy* can be used to perform *adaptive hypothesis testing*. This substantially extends the existing connection between differential privacy and *max-information*, which previously was only known to hold for pure differential privacy. It also extends our understanding of max-information as a partially unifying measure controlling the generalization properties of adaptive data analyses.

Approximate Differential Privacy $\Rightarrow$ Max-Information $\Rightarrow$ Post-selection Hypothesis Testing

Adaptive Data Analysis

- A lot of existing theory assumes tests are selected independently of the data.
- In practice, data analysis is inherently interactive, where experiments may depend on previous outcomes from the same dataset.
- **Question:** How can we provide statistically valid answers to adaptively chosen analyses?
- **Answer:** Limit the information learned about the dataset. [DFH+15a]
- Part of a line of work initiated by [DFH+15a, DFH+15b,HU14].





Post-Selection Hypothesis Testing

- Hypothesis test: Defined by a null hypothesis H_0 and a test statistic t.
- Purpose: Reject H_0 if the data X is not likely to have been generated from some distribution Q^n such that $Q \in H_0$.
- Significance level of $t = \alpha \Longrightarrow \Pr_{X\sim Q^n}[t(X) = Reject] \leq \alpha.$
  - Assumes choice of t is independent of the data X.
- Goal: For an adaptively chosen test $t_{A(X)}$, we want to bound $\Pr_{X\sim Q^n}\left[t_{A(X)}(X) = Reject\right]$ for $Q \in H_0$.
  - Problem: $t_{A(X)}$ can be tailored specifically to X.

Max-Information [DFH+15b]

- An algorithm A with bounded max-info allows the analyst to treat the output $A(X)$ as if it is independent of data X up to a factor.

$$I_\infty^\beta(X; A(X)) \leq k \Leftrightarrow \Pr_{(x,a)}\left(\log\left(\frac{\Pr[(X=x, A(X)=a)]}{\Pr[X'=x]\Pr[A(X)=a]}\right) > k\right) \leq \beta$$

- Differentiate between general and product distributions:

$$I_\infty^\beta(A,n) = \sup_{S:X\sim S} I_\infty^\beta(X; A(X))$$

$$I_{\infty,\Pi}^\beta(A,n) = \sup_{P:X\sim P^n} I_\infty^\beta(X; A(X))$$

- [RRST.16]: If $I_{\infty,\Pi}^\beta(A,n) \leq k$, then for $\gamma(\alpha) = \frac{\alpha-\beta}{2^k}$,

Significance level of $t_{A(X)} = \gamma(\alpha) \Longrightarrow \Pr_{X\sim Q^n}\left[t_{A(X)}(X) = Reject\right] \leq \alpha.$

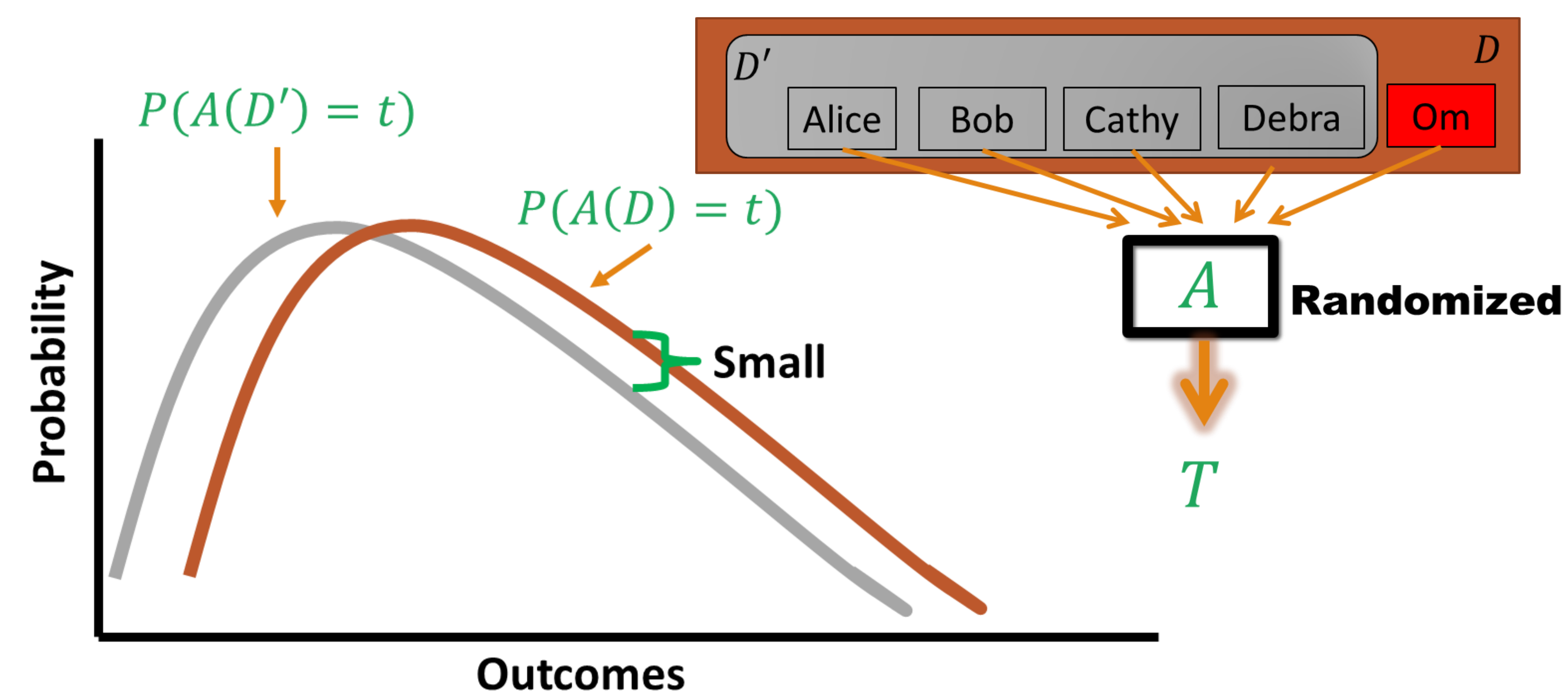
Max-Information $\Rightarrow$ Post-selection Hypothesis Testing

Differential Privacy [DMNS06]

- A randomized algorithm $A: D^n \to T$ is (ε, δ)-differentially private if for all neighboring data sets $x, y \in D^n$, i.e., $dist(x,y) = 1$, and for all sets of outcomes $S \subseteq T$, we have

$$P(A(x) \in S) \leq e^\varepsilon P(A(y) \in S) + \delta$$

  - If δ=0, we say pure DP. If δ>0, we say approximate DP.



Technical Contributions

- Previous results [DFH+15a] : If $A: D^n \to T$ is $(\epsilon, 0)$-DP,
  - For $\beta > 0$, we have $I_{\infty,\Pi}^\beta(A,n) \leq \tilde{O}(\epsilon^2 n)$
  - $I_\infty^0(A,n) \leq \epsilon n$
- **Positive Result:** If $A: D^n \to T$ is (ϵ,δ)-DP, for $\beta \approx O\left(n\sqrt{\delta/\epsilon}\right)$,
  - we have $I_{\infty,\Pi}^\beta(A,n) = O\left(\epsilon^2 n + n\sqrt{\delta/\epsilon}\right)$

Approx. DP $\Rightarrow$ Max-Information

- Consequences:
  - k rounds of adaptivity: max-information $\sim k$ rather than k^2
  - Generalizes and unifies previous work
- **Negative Result:** $\exists$ an (ϵ,δ)-DP algorithm A s.t. $I_\infty^\beta(A,n) \approx n$ for any $\beta \leq \frac{1}{2} - \delta$.

Related Publications

- [BNS+16] Raef Bassily, Kobbi Nissim, Adam D. Smith, Thomas Steinke, Uri Stemmer, and Jonathan Ullman. In STOC, 2016.
- [DFH+15a] Cynthia Dwork, Vitaly Feldman, Moritz Hardt, Toni Pitassi, Omer Reingold, and Aaron Roth. In NIPS. 2015.
- [DFH+15b] Cynthia Dwork, Vitaly Feldman, Moritz Hardt, Toniann Pitassi, Omer Reingold, and Aaron Leon Roth. In STOC, 2015.
- [DMNS06] Cynthia Dwork, Frank Mcsherry, Kobbi Nissim, Adam Smith. In TCC, 2006.
- [HU14] Moritz Hardt and Jonathan Ullman.In FOCS, 2014.
- [RZ16] Daniel Russo and James Zou. In AISTATS, 2016.

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